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SLIP EFFECT AND FLOW FRICTION IN AN ADIABATIC VAPOUR–LIQUID MIXTURE FLOWING IN TUBES

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Аннотация—Анализируются экспериментадьные данные по истинным объемым паросодержаниям и гидравдиьескому сопротивдению при адабатическом движенни парожидкостного потока. Приводятся расчетные рекомендации для определения φ и сопротивления трения.

NOMENCLATURE

- w_0', w_0'', reduced phase velocities (single prime refers to liquid phase; double prime to vapour phase);
- β, $= w_0''/(w_0' + w_0'')$, volumetric vapour content;
- φ, actual volumetric vapour content;
- z, distance between the entry of vapour–liquid mixture to the straight portion of the tube and the given section;
- d, tube diameter;
- w', actual velocity of liquid;
- w_0, circulation rate;
- ν, kinematic viscosity;
- x, vapour content by weight;
- $w\rho$, mass rate;
- ξ, coefficient of flow friction;
- ΔP, pressure difference in vapour–liquid mixture;
- ΔP_f, irreversible portion of pressure difference;
- ΔP_a, reversible portion of pressure difference;
- ΔP_0, pressure difference in liquid phase flow with mass rate $w_0\rho$.

NON-LINEAR pressure distribution along a tube in an adiabatic two-phase flow shows that a vapour–liquid mixture behaves as a compressible fluid. By the character of the change of the axial pressure gradient a two-phase flow may be compared with a single phase-flow, the velocity of which is commensurable with the sonic velocity. In both cases it is impossible to single out, in the strict sense of the word, a hydrodynamically stabilized flow region. However, as compared to a single-phase medium, more strong changes take place along the tube in a two-phase flow as a result of thermal and mechanical interactions between phases as well as between separate elements of each phase (merging and dispersion of bubbles and drops).

The downstream expansion of vapour and the increase of its mass flow rate due to self-evaporation of liquid produces an accelerated mixture flow in which the vapour phase gains greater acceleration than the liquid, i.e. the slip effect arises. In fact, this effect should be most noticeable at low pressures when a slight increase in the mass flow rate of vapour leads to a considerable increase in a volumetric vapour content.

The relative vapour phase velocity appears mainly due to the action of two forces: the buoyancy (Archimedian) force and the pressure force. In turbulent flow a dominating influence is exerted by the force caused by the pressure gradient, which is proved by practically equal values of the actual volumetric content φ obtained both in vertical and horizontal tubes. Under the action of one and the same force determined by the axial pressure gradient, equal elementary volumes of vapour and liquid gain various accelerations due to the difference in masses of these volumes. With the increase of the bubble volume, the dynamic action of the liquid phase upon it also increases, and this in turn produces an increase of its relative velocity.

The relative velocity of the vapour phase must, therefore, increase in the direction of the two-phase flow. As was shown in [1] the value z/d can characterize the effect of the pressure difference on the relative velocity of a bubble. Here z is the distance between the given tube section and the entry of a vapour–liquid mixture into the straight portion of the tube of the given diameter, and d is the diameter of the tube.

However in addition to the above, processes are developing in the flow which counteract the increase of the slip effect. These are, for example, the processes of liquid dispersion and its entrainment into the vapour flow by turbulent fluctuations. With a given mass flow rate the intensity of these processes increases with increase of volumetric flow rate of vapour phase, i.e. with the increase of the ratio between the reduced velocities of the phases.

The recent experimental local values of φ allow the effect of the parameters w_0''/w_0' and z/d on this value to be observed. In Fig. 1(a) the experimental data for φ [2–4] are plotted as φ/β versus w_0''/w_0'.

It is evident that the ratio φ/β can describe the relative velocity of the vapour phase. As seen from the graph, the value φ/β does not depend upon the pressure and the speed of circulation within a wide range of their variations. The division of the curves with respect to the parameter z/d is distinct. With equal values of w_0''/w_0' the actual volumetric vapour content is the lower, the greater the distance between the straight entry of the tube and the place where φ is being determined. For each fixed value of z/d the relative velocity increases from zero ($\varphi/\beta = 1$) to the maximum and then diminishes to zero again (at $\beta \to 1$).

The minima on the curves $\varphi/\beta = f(w_0''/w_0')$ can be probably explained as follows. The sizes of vapour bubbles depend on the volumetric vapour content β. For small values of β the vapour phase exists in a flow in the form of small bubbles and great forces are needed to develop their relative velocities. With an increase of β bubbles grow larger, become deformed more easily which leads to an increase of the relative velocity. For large values of $w_0''/w_0'(\beta)$ the turbulence of flow greatly increases, the liquid phase disperses and is entrained into the vapour flow, and the relative velocity of the flow decreases. It is seen from Fig. 1(a) that with each value of z/d the range of w_0''/w_0' variations [from 0 to (w_0''/w_0')] can be distinguished within which the relative velocity is zero ($\varphi/\beta = 1$). Since a definite value of β corresponds to the gives value of w_0''/w_0', one can say that at $\beta < \beta^*$ the actual volumetric vapour content $\varphi = \beta$. In Fig. 2, the approximate values of β^* are plotted versus z/d. On the same graph the values of β^{**} are presented which correspond to the maximum relative velocity $(\varphi/\beta)_{\min}$. When z/d is changing from 100 to 655. β^* changes slightly, viz. from 0·4 to 0·5. It may therefore be concluded that the greatest deviation of φ from β should be expected within the above range of β variations, i.e. from 0·4 to 0·5.

According to references [2–4] the following calculation formula can be recommended for the definition of the actual volumetric vapour content

$$\varphi = A\left[\frac{w_0''}{w_0'} : \left(\frac{z}{d}\right)^{0\cdot 4}\right]^n \tag{1}$$

The values of A and n are given in Table 1. Formula (1) is valid for $z/d > 50$.

It should be noted that in practice the vapour–mixture is not made artificially in a mixer or by choking, but is produced directly when vapour bubbles are formed on the surface of heated tubes.

In Fig. 1(b) the plotted values of φ/β are those obtained in papers [5, 6]. As a heat carrier Freon 12 [5] and water [6] were used. With equal relative distances z/d from the heated portion to the place of translucency the data for Freon 12 and vapour–water mixture agree satisfactorily.

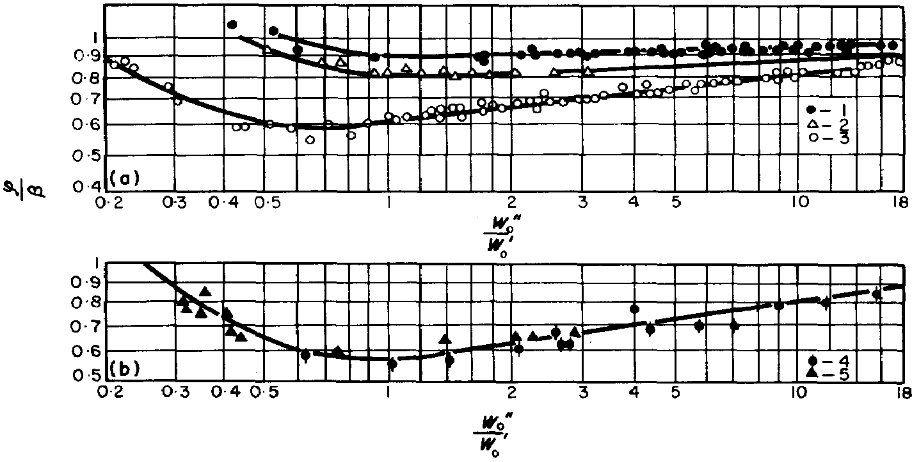

FIG. 1. φ/β versus the ratio of reduced phase velocities w_0''/w_0'.
1.—vertical tube [2], $z/d \approx 55$, $P = 19{\cdot}6$–$68{\cdot}6$ bar, $w\rho = 400$–3000 kg/m^{2}s;
2.—vertical tube [3], $z/d \approx 77$, $P = 19{\cdot}6$–$78{\cdot}4$ bar, $w\rho = 570$–850 kg/m^{2}s;
3.—horizontal tube [4], $z/d \approx 655$, $P = 1{\cdot}96$–$2{\cdot}94$ bar, $w\rho = 500$–1100 kg/m^{2}s;
4.—vertical tube [5]; $z/d \approx 20$;
5.—vertical tube [6]; $z/d \approx 20$.

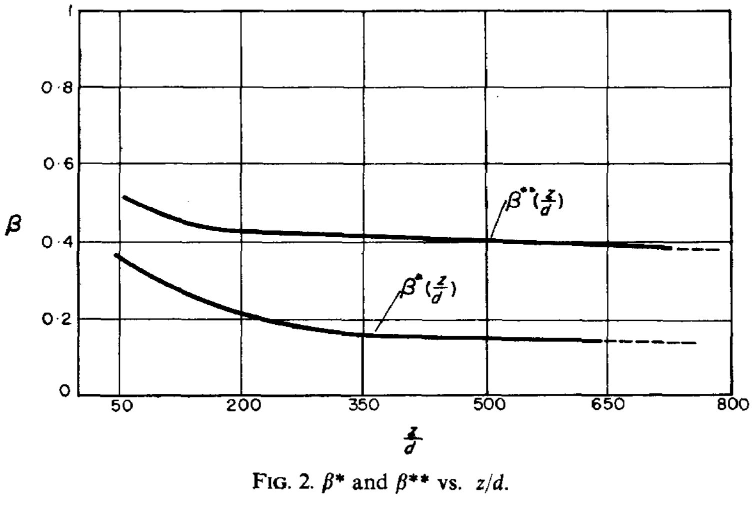

FIG. 2. β^* and β^{**} vs. z/d.

Table 1

$\frac{w_0''}{w_0'}\cdot\left(\frac{x}{d}\right)^{-0\cdot4}$	n	A
0·03–0·14	0·6	1·42
0·14–0·7	0·333	0·84
0·7 –4	0·12	0·77
4 –40	0·04	0·85
>40	0	1

Under these conditions, for the same values of w_0''/w_0' the relative velocity of vapour is greater than in the case when adiabatic vapour–liquid mixture obtained by choking or mixing is flowing in front of the section where the actual vapour content φ is determined (Fig. 1a). This can be explained by the fact that in tubes with boiling the friction is essentially greater and, therefore, the vapour phase gains greater acceleration under the action of forces which are due to the large pressure gradient.

The friction in the flow of the vapour–liquid mixture is directly determined by the actual vapour content of the flow. Extreme complicacy of the phenomenon impedes the analysis even one based on a one-dimensional model. Therefore the theoretical analysis of rather abstracted schemes is of importance, as it might clarify the principal laws of a two-phase mixture flow. Such attempts were made earlier [7–9].

Since across a given section of the tube the pressure has the same value, then over the tube length l the pressure difference in the two-phase flow will be equal to the pressure difference in each phase

$$\Delta P = \Delta P' = \Delta P'' \tag{2}$$

The total pressure difference is the sum of its reversible and irreversible components

$$\Delta P = \Delta P_f + \Delta P'_a + \Delta P''_a \tag{3}$$

$$\Delta P' = \Delta P'_f + \Delta P'_a \tag{4}$$

In adiabatic two-phase flow the acceleration of the vapour phase can be neglected, and then the ratio of the pressure difference due to the friction in the two-phase flow to that in liquid flow with the same mass rate ΔP_0 at the temperature of saturation can be expressed in the form

$$\frac{\Delta P}{\Delta P_0} = \frac{\Delta P'_f}{\Delta P_0} \tag{5}$$

If the fraction of the tube section filled with vapour is φ, and that with liquid $(1 - \varphi)$, then the diameter equivalent to the section through which vapour flows will be $d\sqrt{\varphi}$, and that for liquid, $d\sqrt{(1 - \varphi)}$ assuming that vapour and liquid flow through tubes of circular cross-sectional areas $\varphi(\pi/4)d^2$ and $(1 - \varphi)(\pi/4)d^2$, respectively. Since $\Delta P'_f$ and ΔP_0 are proportional to the kinetic energy of the flow along the lengths corresponding to the diameters, then

$$\frac{\Delta P}{\Delta P_0} = \frac{\xi' w'^2}{\xi_0 w_0^2} \tag{6}$$

with

$$\xi' = \frac{0{\cdot}3164 \nu'^{0{\cdot}25}}{[w'd\sqrt{(1 - \varphi)}]^{0{\cdot}25}} \tag{7}$$

$$\xi_0 = \frac{0{\cdot}3164 \nu'^{0{\cdot}25}}{(w_0 d)^{0{\cdot}25}} \tag{8}$$

we obtain from (6)

$$\frac{\Delta P}{\Delta P_0} = \frac{(1 - x)^{1{\cdot}75}}{(1 - \varphi)^{1{\cdot}875}} \tag{9}$$

In (9) x is an average, over the length l, actual vapour content of the flow by weight, and φ the actual vapour content of the flow by volume.

Relation (9) is valid for vertical and horizontal tubes. Since the average actual vapour content does not refer to some particular flow structure, equation (9) practically does not depend on the flow pattern.

In Fig. 3, experimental data from references [10] and [11] are plotted. Though all the parameters in the experiment are equal, the values of $\Delta P/\Delta P_0$ differ more than 25 per cent. This can be explained by the difference of the distances between the measuring section and the entry of the vapour–liquid mixture to the tube of the prescribed diameter.

If in equation (9) φ is determined from formula (1) where z/d is the distance from the entry to the straight portion of the tube to the middle of the measuring portion, then satisfactory agreement is obtained between the experimental and predicted values of $\Delta P/\Delta P_0$.

In Fig. 4, experimental data of fluid friction obtained by various authors [10–12] are plotted against

$$\frac{(1 - x)^{1{\cdot}75}}{(1 - \varphi)^{1{\cdot}875}}$$

As seen the scatter of the experimental points does not exceed ± 20 per cent under the above treatment within the given range of variations of pressure and mass rates.

When adiabatic vapour–liquid mixture flows in very long tubes, the fluid friction must be determined for separate portions.

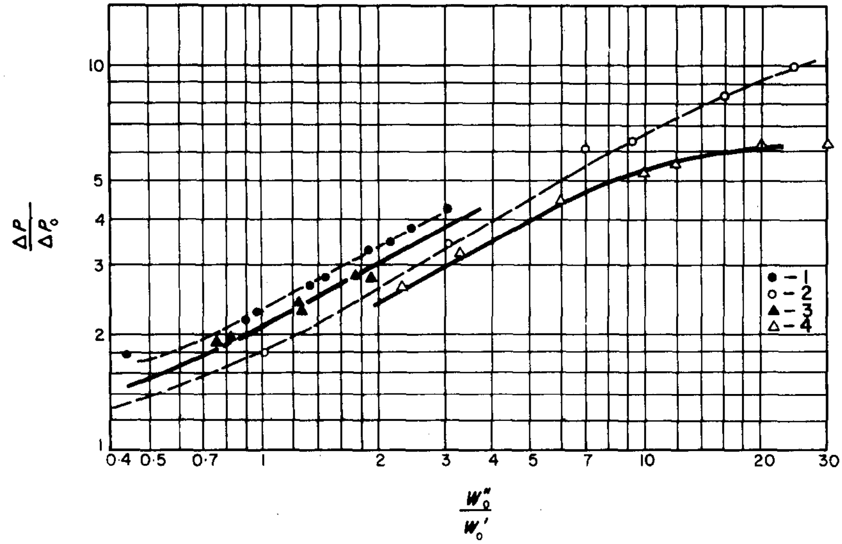

FIG. 3. $\Delta P/\Delta P_0$ versus the ratio of reduced phase velocities w_0''/w_0'. (P = 98 bar).
$w\rho$ = 1000 kg/m^{2}s; 1.— $z/d \approx$ 87·5 [11]; 2.— $z/d \approx$ 150 [10];
$w\rho$ = 2000 kg/m^{2}s; 3.— $z/d \approx$ 87·5 [11]; 4.— $z/d \approx$ 110 [10].

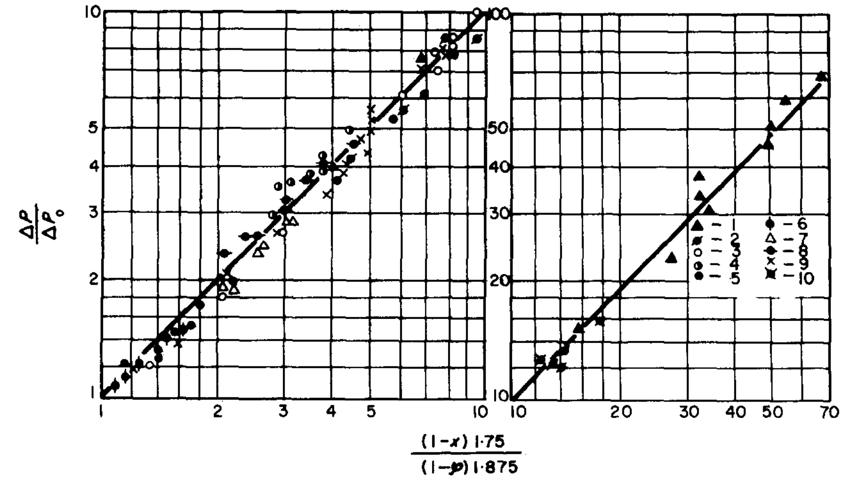

FIG. 4. Generalized relation between $\Delta P/\Delta P_0$ and $(1-x)^{1\cdot75}/(1-\varphi)^{1\cdot875}$
Reference [10]: 1.—P = 25·5 bar, $w\rho$ = 800 kg/m^{2}s;
2.—P = 98 bar, $w\rho$ = 700 kg/m^{2}s;
3.—P = 98 bar, $w\rho$ = 1000 kg/m^{2}s;
4.—P = 166 bar, $w\rho$ = 800 kg/m^{2}s.
Reference [11]: 5.—P = 147 bar, $w\rho$ = 1000 kg/m^{2}s;
6.—P = 196 bar, $w\rho$ = 2000 kg/m^{2}s;
7.—P = 98 bar, $w\rho$ = 2000 kg/m^{2}s;
8.—P = 98 bar, $w\rho$ = 500 kg/m^{2}s.
Reference [12]: 9.—P = 98 bar, $w\rho$ = 3000 kg/m^{2}s.
Authors' data 10.—P = 1·47 bar, $w\rho$ = 1400 kg/m^{2}s.

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Abstract—Experimental data on actual volumetric vapour contents and hydrodyanmic friction are analysed in an adiabatic flow of a vapour–liquid mixture. Calculation procedures are given for determining φ and the friction drag.

Résumé—Les résultats expérimentaux pour les teneurs volumiques réelles en vapeur et le frottement hydrodynamique sont analysés dans le cas d'un mélange de vapeur et de liquide. On expose les détails des calculs pour déterminer φ et la traînée de frottement.

Zusammenfassung—Versuchswerte für den tatsächlichen volumetrischen Dampfgehalt und die hydrodynamische Reibung sind für die adiabate Strömung eines Dampf-Flüssigkeitsgemisches analysiert. Zur Bestimmung von φ und dem Reibungswiderstand sind Berechnungsverfahren angegeben.